Electric potential due to a washer

Solution A (Invented by me.)

Follow the reasoning of LSM example 7.15, "Potential Due to a Uniform Disk of Charge" (pages 305–306) but at the last equation integrate from $R/5$ to R instead of from 0 to R:

$$\begin{aligned}
V &= k_e\, 2\pi\sigma \int_{R/5}^{R} \frac{r\,dr}{\sqrt{z^2+r^2}} \qquad [\![\text{integrate by substitution using } u = z^2+r^2]\!] \\
&= k_e\, 2\pi\sigma \int_{z^2+(R/5)^2}^{z^2+R^2} \frac{du/2}{u^{1/2}} \\
&= k_e\, 2\pi\sigma \left[u^{1/2}\right]_{z^2+(R/5)^2}^{z^2+R^2} \\
&= k_e\, 2\pi\sigma \left(\sqrt{z^2+R^2} - \sqrt{z^2+(R/5)^2}\right).
\end{aligned}$$

Solution B (Invented by Janelle Reyes, class of 2000... much more elegant than my solution!)

This situation is the same as a disk of radius R and charge density $+\sigma$, PLUS a disk of radius $R/5$ and charge density $-\sigma$. The electric potentials due to such disks are given by LSM as the last equation of example 7.15, "Potential Due to a Uniform Disk of Charge" (pages 305–306):

$$\begin{aligned}
V &= \left[\text{potential due to disk of radius } R \text{ and charge density } +\sigma\right] \\
&\quad + \left[\text{potential due to disk of radius } R/5 \text{ and charge density } -\sigma\right] \\
&= \left[k_e\, 2\pi(+\sigma)\left(\sqrt{z^2+R^2} - z\right)\right] + \left[k_e\, 2\pi(-\sigma)\left(\sqrt{z^2+(R/5)^2} - z\right)\right] \\
&= k_e\, 2\pi\sigma \left(\sqrt{z^2+R^2} - \sqrt{z^2+(R/5)^2}\right).
\end{aligned}$$

Grading: As you can see, there are many ways to solve this problem. I did the integral by hand, but anyone who does it using a handbook or a computer algebra system should also get full credit. In general, there should be 3 points for setup and 7 points for executing that setup, but recognize that there are many different correct ways to setup and to execute.