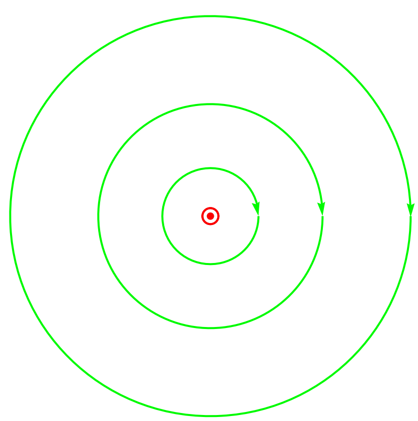
What is the magnetic field produced by a curtain of current?

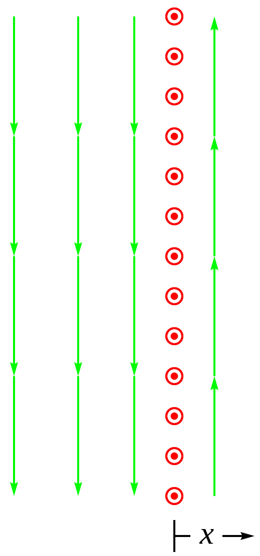
More specifically: Suppose we had a large number of parallel wires, as in ribbon cable (also called "jumper wires"):

Let's think of an infinite number of such wires, each straight and infinitely long. (We don't go half-way in this course.) Call this a curtain of current. Each wire carries current $i$, and there are $n$ wires per meter.

One infinitely long straight wire sets up magnetic field like this:

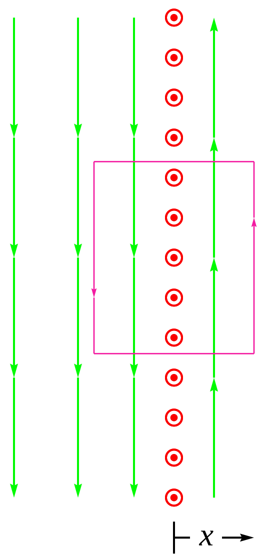


So a curtain sets up magnetic field like this:




It is clear from symmetry that the magnetic field must point straight up or straight down, and its magnitude can depend only upon the distance $x$ from the curtain. (That last fact comes from symmetry under a 180° rotation about an axis pointing out of the page.)

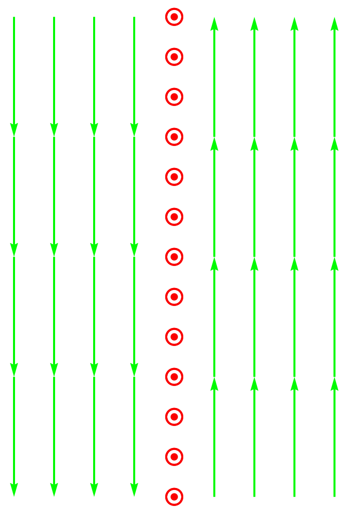
Now we know the qualitative character of the magnetic field. What is its magnitude $B(x)$? We find this by applying Ampere's law to the purple loop below:



We find

$$\begin{aligned}\oint \vec{B} \cdot d\vec{\ell} &= \mu_0 I_{\text{enclosed}} \\ 2B(x)(\text{height of loop}) &= \mu_0 in(\text{height of loop}) \\ B(x) &= \tfrac{1}{2}\mu_0 ni \qquad (1)\end{aligned}$$

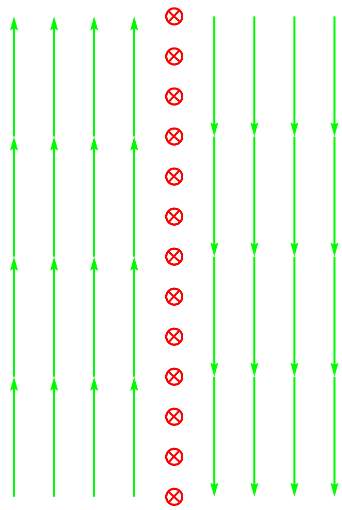
I am surprised that this magnetic field is in fact independent of $x$... I would have expected it to decrease with $x$. But of course when we play with infinity surprises are bound to happen. Below is the figure for the field as it actually is, independent of $x$:



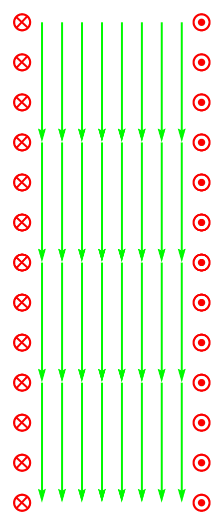
Now, what happens if there are two parallel curtains, the one above with current flowing out of the page, then another with current flowing into the page?

We approach this problem using rotational symmetry to find the field of the curtain with current flowing into the page, followed by superposition to find the field due to both curtains.

Rotate the above current and field configuration by 180° about an axis pointing up the page. The result is



Now, superpose these two configurations. To the right of both curtains, the fields exactly cancel ("destructive interference"). To the left of both curtains, the fields exactly cancel. But between the two curtains, the fields double ("constructive interference").



Because of the doubling, the magnetic field between the two curtains has magnitude $\mu_0 in$. (You might remember that this is the formula for the magnetic field within a solenoid. And indeed, this is a solenoid of rectangular cross section with the long side of the rectangle infinitely long.)