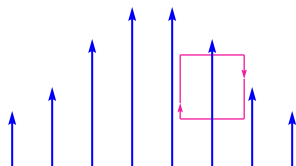


Circularly polarized standing waves

Question: For a circularly polarized standing wave of light, is the magnetic field parallel to or perpendicular to the electric field?

Setup

The electric field turns like a jump rope. Here's a snapshot of the electric field (blue) at an instant.

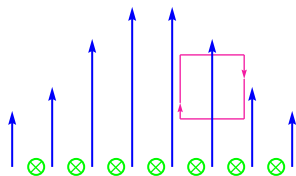


Apply Faraday's Law

$$\oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$$

to the magenta-colored loop. The line integral on the left is non-zero. But what is Φ_B , the flux of magnetic field through the surface bounded by the magenta-colored loop? How does that flux change with time?

\vec{B} perpendicular to \vec{E} scenario

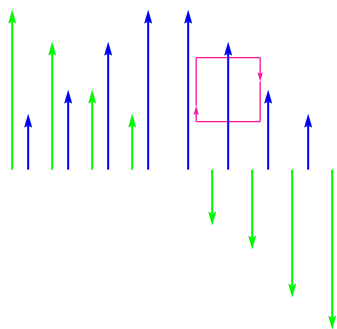


Here the flux Φ_B is positive, and in fact it's at a maximum. As the blue "jump rope" rotates into the page, the magnetic field (green) follows, so the magnetic flux through the stationary magenta-lined loop decreases.

Now, because Φ_B is at a maximum, $d\Phi_B/dt$ is zero. Under this possibility, the left hand-side of Faraday's Law is non-zero, while the right-hand side is zero.

Bad idea.

\vec{B} parallel to \vec{E} scenario



Here the flux Φ_B is zero, but as the fields rotate, the flux changes. Under this possibility, the left hand-side of Faraday's Law is non-zero, while the right-hand side is also non-zero.

We haven't proven this possibility to be correct, but it's not impossible, whereas the " \vec{B} perpendicular to \vec{E} scenario" is impossible. (A more technical analysis shows that it is indeed correct.)

Here is the *more technical analysis*, suitable for students who have studied vector calculus: Start with the differential form of Faraday's Law

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}.$$

For a circularly polarized standing wave,

$$\vec{E} = E_0 \sin(kx)[\pm \hat{y} \sin(\omega t) + \hat{z} \cos(\omega t)],$$

where the \pm depends on whether the \vec{E} rotates clockwise or counterclockwise. From this it follows that

$$\vec{\nabla} \times \vec{E} = E_0 k \cos(kx)[- \hat{y} \cos(\omega t) \pm \hat{z} \sin(\omega t)]$$

whence

$$\frac{\partial \vec{B}}{\partial t} = E_0 k \cos(kx)[\hat{y} \cos(\omega t) \mp \hat{z} \sin(\omega t)].$$

Integrating with respect to t ,

$$\begin{aligned} \vec{B} &= (E_0/c) \cos(kx)[\hat{y} \sin(\omega t) \pm \hat{z} \cos(\omega t)] \\ &= \pm (E_0/c) \cos(kx)[\pm \hat{y} \sin(\omega t) + \hat{z} \cos(\omega t)], \end{aligned}$$

where the upper sign is shown (inadequately) in the sketch above.