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Double branched covers of theta-curves

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ABSTRACT

We prove a folklore theorem of Thurston, which provides necessary and sufficient conditions for primality of a certain class of theta-curves. Namely, a theta-curve in the 3-sphere with an unknotted constituent knot κ is prime, if and only if lifting the third arc of the theta-curve to the double branched cover over κ produces a prime knot. We apply this result to Kinoshita's theta-curve.

Keywords: Theta-curve; prime; double branched cover; equivariant Dehn lemma; Kinoshita's theta-curve.

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1. Introduction

Consider the multigraph Γ with two vertices v_1, v_2 and three edges e_1, e_2, e_3 none of which are loops. A *theta-curve* is a locally flat embedding of Γ in S^3 or in $\mathbb{R}^3$. Each theta-curve θ has three *constituent knots*: $e_2 \cup e_3$, $e_1 \cup e_3$ and $e_1 \cup e_2$. Given a constituent knot κ, there is exactly one arc e of θ not contained in κ.

A theta-curve is *unknotted* provided it lies on an embedded S^2 and is *knotted* otherwise. We will use two operations on knots and theta-curves: the order-2 connected sum $\#_2$ and order-3 connected sum $\#_3$. An order-2 connected sum $\theta \#_2 J$ of a theta-curve $\theta \subset S^3$ and a knot $J \subset S^3$ is the result of deleting unknotted ball-arc pairs from each of (S^3, θ) and (S^3, J), and then identifying the resulting boundary spheres. The ball-arc pair in (S^3, θ) must be disjoint from the vertices

v_1 and v_2. An order-3 connected sum $\theta_1 \#_3 \theta_2$ of two theta-curves is the result of deleting an unknotted ball-prong neighborhood of a vertex from each theta-curve, and then identifying the resulting boundary spheres. Each of these operations yields a theta-curve in S^3.

Remark 1.1. Wolcott has shown that the order-3 connected sum is independent of the glueing homeomorphism provided one specifies the vertices at which to sum and the pairing of the arcs [9]. The operations $\theta \#_2 J$ and $\theta_1 \#_3 \theta_2$ are ambiguous as presented. The former could mean up to six different theta-curves and the latter could mean up to 24 different theta-curves. In each instance below, this ambiguity is either irrelevant or is sufficiently eliminated by context.

A theta-curve θ is *prime* provided the following three conditions are satisfied: (i) θ is knotted, (ii) θ is not an order-2 connected sum of a nontrivial knot and a (possibly unknotted) theta-curve and (iii) θ is not an order-3 connected sum of two knotted theta-curves. We adopt the convention that the unknot is not prime.

Let $S^3 \subset \mathbb{R}^4$ be the unit sphere. Let $g \in \mathrm{SO}(4)$ denote the orientation preserving involution of S^3, whose matrix is diagonal with entries $[-1, -1, 1, 1]$. Note that, $\mathrm{Fix}(g)$ is a great circle in S^3 and is therefore unknotted. Let $G = \{e, g\}$ be the group of order two. Throughout this paper, equivariance is with respect to G. Recall that a subset $D \subset S^3$ is *equivariant* provided $g(D) = D$ (setwise) or $g(D) \cap D = \emptyset$.

Suppose the theta-curve θ has an unknotted constituent knot κ and $\theta = e \cup \kappa$. By an ambient isotopy, we can assume $\kappa = \mathrm{Fix}(g)$. Let $b : S^3 \to S^3$ be the double branched cover with branch set $\mathrm{Fix}(g)$ and such that $b \circ g = b$. Lifting the arc e of θ yields the knot $K := b^{-1}(e)$ in S^3. We call K the *lifted knot of θ with respect to κ*. By stereographic projection from $(0, 0, 0, 1) \in S^3$, the map g descends to the rotation of $\mathbb{R}^3$ about the z-axis by half a revolution.

Moriuchi attributes the following theorem to Thurston without proof [7, Proposition 4.1], and Moriuchi references an unpublished letter of Litherland for Thurston's statement of this theorem [8, Proposition 5.1].

Main Theorem (W. Thurston). *Suppose θ has an unknotted constituent knot κ, and let K be the lifted knot of θ with respect to κ. The theta-curve θ is prime, if and only if the lifted knot K is prime.*

Our proof of the Main Theorem uses the equivariant Dehn Lemma in two places. In the proof of Lemma 2.3, Kim and Tollefson's version [5, Lemma 3] suffices as noted by the referee. In the proof of Lemma 2.7, we use Edmond's version [3]. It's possible that with some tinkering, Kim and Tollefson's version suffices.

In the final section of this paper, we use the Main Theorem to prove that Kinoshita's theta-curve is prime. We also explain how primality of Kinoshita's theta-curve yields an alternate proof of the irreducibility of certain tangles.

2. Proof of Main Theorem

We will prove the contrapositive in both directions. First, we observe two useful lemmas.

Lemma 2.1. *Suppose θ, θ_1 and θ_2 are theta-curves, and $J \subset S^3$ is a nontrivial knot.*

(2.1) *If $\theta \#_2 J$ has an unknotted constituent knot κ, then $\kappa \subset \theta$ (that is, κ is the union of two edges in θ). Let K be the lifted knot of θ with respect to κ. Then, the lifted knot of $\theta \#_2 J$ with respect to κ is $K \# J \# J$.*

(2.2) *If $\theta_1 \#_3 \theta_2$ has an unknotted constituent knot κ, then there are unknotted constituent knots $\kappa_1 \subset \theta_1$ and $\kappa_2 \subset \theta_2$ such that the order-3 connected sum $\theta_1 \#_3 \theta_2$ induces the connected sum $\kappa = \kappa_1 \# \kappa_2$. Let K_j be the lifted knot of θ_j with respect to κ_j for $j \in \{1, 2\}$. Then, the lifted knot of $\theta_1 \#_3 \theta_2$ with respect to κ is $K_1 \# K_2$.*

Proof. For the first claim in (2.1), label the edges of θ, so that the sum $\theta \#_2 J$ is performed along $e_3 \subset \theta$ and let e be the resulting edge in $\theta \#_2 J$. The knot $e_1 \cup e$ is the connected sum of the knots $e_1 \cup e_3$ and J. As J is nontrivial and nontrivial knots, do not have inverses under connected sum, we see that the knot $e_1 \cup e$ is nontrivial. Similarly, the knot $e_2 \cup e$ is nontrivial. Hence, $e_1 \cup e_2$ must be an unknot as desired. The first claim in (2.2) follows similarly.

The remaining claims follow from the definitions of order-2 and order-3 connected sum and from the definition of the double branched cover $b : S^3 \to S^3$. □

For the remainder of this section, we assume θ is a theta-curve with unknotted constituent knot $\kappa = \mathrm{Fix}(g)$ and $\theta = e \cup \kappa$. Let K be the lifted knot of θ with respect to κ.

Lemma 2.2. *If Σ is an equivariant 2-sphere in S^3, that meets κ in exactly two points, then $b(\Sigma)$ is an embedded 2-sphere in S^3 transverse to κ and meeting κ in exactly two points.*

Proof. As Σ meets $\kappa = \mathrm{Fix}(g)$, equivariance implies $g(\Sigma) = \Sigma$. Equivariance also implies that Σ is transverse to κ, $b(\Sigma)$ is a closed connected surface, and $b(\Sigma)$ is transverse to κ. Let Σ' be the equivariant annulus obtained from Σ by deleting the interiors of disjoint equivariant 2-disk neighborhoods of the two points $\Sigma \cap \kappa$. The restriction of b to $\Sigma' \to b(\Sigma')$ is a double cover and $b(\Sigma)$ is obtained from $b(\Sigma')$ by glueing in two 2-disks. It follows that the Euler characteristic of $b(\Sigma')$ is 0 and the Euler characteristic of $b(\Sigma)$ is 2. Hence, $b(\Sigma)$ is a 2-sphere. □

Lemma 2.3. *The theta-curve θ is unknotted, if and only if K is unknotted.*

Proof. Clearly, if θ is unknotted, then K is unknotted.

Suppose K is unknotted. Let N be a closed regular equivariant neighborhood of K in S^3. As $S^3 - \operatorname{Int} N$ is a solid torus, it has compressible boundary. By the equivariant Dehn Lemma [5, Lemma 3], there exists a properly embedded equivariant compressing disk $D \subset S^3 - \operatorname{Int} N$. If $g(D) = D$, then a slight pushoff D' of D is an equivariant compressing disk with $g(D') \cap D' = \emptyset$ and we may redefine D to be D' instead. Hence, we may assume $D \cap g(D) = \emptyset$. As $D \cap N = \partial D$ is a longitude of the solid torus N, there exists an embedded annulus $A \subset N$, such that $\partial A = K \cup \partial D$ and $A \cap g(A) = K$. The 2-sphere $\Sigma = D \cup A \cup g(A) \cup g(D)$ contains K, is transverse to $\kappa = \operatorname{Fix}(g)$, and meets κ in exactly two points. Therefore, Σ divides (S^3, κ) into two equivariant unknotted ball-arc pairs. Each of these balls contains an equivariant 2-disk with boundary K and containing that balls arc of κ. The union of these two disks is a new 2-sphere Σ', such that $b(\Sigma')$ is a sphere containing θ as desired. □

To prove the reverse implication of the Main Theorem, suppose θ is not prime. If θ is unknotted, then K is the unknot which is not prime. If $\theta = \theta_0 \#_2 J$ and J is nontrivial, then, by Lemma 2.1, $K = K_0 \# J \# J$ which is not prime. Otherwise, $\theta = \theta_1 \#_3 \theta_2$, where θ_1 and θ_2 are both knotted. Then, by Lemma 2.1, $K = K_1 \# K_2$ is a sum of knots, which are nontrivial by Lemma 2.3. This proves the reverse implication of the Main Theorem.

To prove the forward implication of the Main Theorem, suppose K is not prime. If K is unknotted, then so is θ by Lemma 2.3. Otherwise, there is a sphere Σ, that splits (S^3, K) into two knotted ball-arc pairs.

Lemma 2.4. *If $\Sigma \cap g(\Sigma) = \emptyset$, then θ is a nontrivial order-2 connected sum. If $\Sigma = g(\Sigma)$ and Σ meets κ at exactly two points distinct from v_1 and v_2, then θ is a nontrivial order-3 connected sum.*

Proof. If $\Sigma \cap g(\Sigma) = \emptyset$, then Σ bounds a ball B disjoint from $g(\Sigma)$. It follows that B is disjoint from $g(B)$. As $\kappa = \operatorname{Fix}(g)$ is connected and disjoint from Σ, κ must also be disjoint from B. Thus $(B, B \cap K)$ maps homeomorphically by b to a nontrivial ball-arc pair in (S^3, θ). Thus, θ is a nontrivial order-2 connected sum.

Suppose $\Sigma = g(\Sigma)$ meets κ at exactly two points distinct from v_1 and v_2. As $g(\Sigma \cap K) = \Sigma \cap K$ and g interchanges the two lifts α and β of e, Σ meets each of α and β once. Therefore, the vertices v_1 and v_2 must lie in different components of $S^3 - \Sigma$, and so Σ meets each arc of κ once. In particular, the G-action does not interchange the balls in S^3 bounded by Σ. By Lemma 2.2, $b(\Sigma)$ is a sphere and it splits θ as an order-3 connected sum. Each of these summands must be nontrivial since Σ splits (S^3, K) into two knotted ball-arc pairs. □

Let Σ be a sphere such that:

(2.1) Σ separates (S^3, K) into two knotted ball-arc pairs.
(2.2) Σ and $g(\Sigma)$ are in general position with each other.
(2.3) $\Sigma \cap g(\Sigma) \cap K = \emptyset$.

Condition (2.2) is achieved by the proof of Lemma 1 from [4, p. 148]. By Lemma 2.4, it suffices to show that, we can either improve Σ (while maintaining (2.1)–(2.3)) and make it disjoint from $g(\Sigma)$, or we can produce a sphere Σ' which bounds two knotted ball-arc pairs in (S^3, K), such that $g(\Sigma') = \Sigma'$ and Σ' meets κ at exactly two points distinct from v_1 and v_2.

Lemma 2.5. *Any curves of $\Sigma \cap g(\Sigma)$, which are inessential in $\Sigma - K$ can be removed without introducing new intersections.*

Proof. Note that a curve in $\Sigma \cap g(\Sigma)$ is essential in $\Sigma - K$, if and only if it is essential in $g(\Sigma) - K$. Consider a component c of $\Sigma \cap g(\Sigma)$, that is inessential in $\Sigma - K$ and is innermost in $g(\Sigma) - K$. Then c bounds closed disks $D_1 \subset g(\Sigma) - K$ and $D_2 \subset \Sigma - K$ and $D_1 \cup D_2$ is an embedded 2-sphere. As D_1 and D_2 are both disjoint from K, $D_1 \cup D_2$ bounds a ball B disjoint from K.

Case 1. $D_1 \cap g(D_1) = \emptyset$. Then, there is a neighborhood N of D_1, such that $N \cap g(N) = \emptyset$ and $N \cap \Sigma \cap g(\Sigma) = c$. Improve Σ by pushing D_2 past D_1 into N using B. Since the only part of Σ, that changed now lies in N and N is now disjoint from $\Sigma \cap g(\Sigma)$, there are no new intersections.

Case 2. $D_1 \cap g(D_1) \neq \emptyset$. Since D_1 is innermost, this means that $c = g(c)$ and $D_1 = g(D_2)$. Using B, push D_2 past D_1 to a parallel copy of D_1. This removes c without adding new intersections. □

By Lemma 2.5, we may assume all components of $\Sigma \cap g(\Sigma)$ are essential in $\Sigma - K$. Let $c_1, c_2, \ldots, c_n$ be the components of $\Sigma \cap g(\Sigma)$ and let $A_{ij} \subset \Sigma$ be the annulus with $\partial A_{ij} = c_i \cup c_j$ for $i \neq j$. We assume c_i and c_{i+1} are adjacent in $\Sigma - K$ for $1 \leq i \leq n-1$. Let $\pi \in \mathrm{Sym}(n)$ be the permutation, such that $g(c_i) = c_{\pi(i)}$. As g is an involution, either π is the identity or π has order two.

Lemma 2.6. *If $\pi(i) = i$ and c_i bounds a disk $D \subset \Sigma$, such that $\mathrm{Int}\, D \cap g(\mathrm{Int}\, D) = \emptyset$, then either c_i can be removed without introducing intersections or θ is a nontrivial order-3 connected sum.*

Proof. Suppose D and i are as indicated. Then, $D \cup g(D)$ is a sphere invariant under g. Since g is orientation preserving, $D \cup g(D)$ must have at least one fixed point under g; in fact, it must have two, because κ meets $D \cup g(D)$ transversely. As $\mathrm{Int}\, D \cap g(\mathrm{Int}\, D) = \emptyset$, all such fixed points must lie in c_i. As Σ is transverse to κ, c_i contains a finite number of fixed points under g. Let $a \subset c_i$ be an arc intersecting κ exactly at its endpoints, so $\kappa \cap a = \partial a$. Then $a \cup g(a)$ is a simple closed curve, so $c_i = a \cup g(a)$ and thus c_i cannot contain more than two fixed points. Thus, if $D \cup g(D)$ bounds two knotted ball-arc pairs, then we are done by Lemma 2.4.

Otherwise, $D \cup g(D)$ bounds an unknotted ball-arc pair $(B, B \cap K)$. Using B, we can push D (and any other components of $\Sigma \cap B$) past $g(D)$ to remove c_i. □

Thus, we must show that as long as $\Sigma \cap g(\Sigma) \neq \emptyset$, there is a curve c_i as in Lemma 2.6.

Lemma 2.7. *Suppose $T \subset S^3$ is a torus, such that $g(T) = T$ and $T \cap \kappa = \emptyset$. Let $Z \subset S^3$ be the half bounded by T containing κ. If $c \subset T$ is essential in T, null-homotopic in Z, and equivariant, then it is invariant (setwise).*

Proof. Suppose T, Z and c are as indicated. Since $\kappa \subset Z$, it is clear that $g(Z) = Z$. As c is equivariant, null-homotopic in Z, and disjoint from κ, c bounds an equivariant disk $D \subset Z$ by the equivariant Dehn Lemma [3].

Suppose, by way of contradiction, that $g(c) \neq c$. Then, as c and D are equivariant, we have $c \cap g(c) = \emptyset$ and $D \cap g(D) = \emptyset$. The set $Z - (D \cup g(D))$ has two components, the closures in Z of these are B_1 and B_2. Each of B_1 and B_2 is bounded by the disks D and $g(D)$ as well as an annulus in T, so both B_1 and B_2 are 3-balls. Now, κ is contained in one of these balls. Without loss of generality, $\kappa \subset B_1$. So, both B_1 and B_2 must be fixed setwise by g. As B_1 is fixed by g and g is orientation preserving, g must have a fixed point on $\partial B_1 \subset T \cup D \cup g(D)$. But, the set of fixed points of g is exactly κ, and T, D and $g(D)$ are all disjoint from κ. This is a contradiction, so c must be invariant. □

In the following lemma, we take $i, j, k, l \in \{1, 2, \ldots, n\}$.

Lemma 2.8. *If $\pi(j) > j$, then there is some $j < i < \pi(j)$, such that $\pi(i) = i$. In particular, there is some i, such that $\pi(i) = i$.*

Proof. Suppose, by way of contradiction, that $\pi(j) > j$ and there is no such i. Choose k such that:

(2.1) $j \leq k < \pi(k) \leq \pi(j)$.
(2.2) $\pi(k) - k$ is minimal such that (2.1) is satisfied.

If $k < l < \pi(k)$, then we cannot have $k < \pi(l) < \pi(k)$. So, $A_{k\pi(k)} \cap g(A_{k\pi(k)}) = c_k \cup c_{\pi(k)}$ and $T = A_{k\pi(k)} \cup g(A_{k\pi(k)})$ is a torus. Furthermore, $T = g(T)$ and T is disjoint from κ. As κ is connected, one component of $S^3 - T$ contains κ. Let Z be its closure in S^3. Since T is also disjoint from K and $K \cup \kappa$ is connected, we have $K \subset Z$. Let m be minimal, such that $c_m \subset T$. Then, c_m bounds a disk in Σ with interior disjoint from T, which intersects K. So, c_m and all other curves in T of the same homotopy type are null-homotopic in Z. In particular, c_k is essential in T, null-homotopic in Z, and equivariant. Hence, c_k is invariant by Lemma 2.7. This implies that $\pi(k) = k$, a contradiction.

The second part is immediate as either $\pi(1) = 1$, or $\pi(1) > 1$ and there is $1 < i < \pi(1)$, such that $\pi(i) = i$. □

Let $i \in \{1, 2, \ldots, n\}$ be minimal, such that $\pi(i) = i$ (this i exists by Lemma 2.8). Let $D \subset \Sigma$ be a disk with $\partial D = c_i$ such that: (i) D contains c_{i-1} in case $i > 1$,

(ii) D does not contain c_{i+1} in case $i < n$ and (iii) D is either disk in Σ bounded by c_1 in case $i = 1$. We claim that $D \cap g(D) = c_i$. Suppose, by way of contradiction, that $c_k \subset D \cap g(D)$. Then, $1 \leq k < i$ and $1 \leq \pi(k) < i$. If $\pi(k) = k$, then k contradicts minimality of i. If $k < \pi(k)$, then Lemma 2.8 yields $k < i' < \pi(k) < i$, such that $\pi(i') = i'$, which contradicts minimality of i. If $\pi(k) < k$, then let $j = \pi(k)$. So, $j = \pi(k) < k = \pi(j)$ since π^2 is the identity. Again, Lemma 2.8 yields $j < i' < \pi(j) < i$, such that $\pi(i') = i'$, which contradicts minimality of i. Hence, the claim $D \cap g(D) = c_i$ holds. By Lemma 2.6, either c_i can be removed or θ is a nontrivial order-3 connected sum. Thus, if θ is not a nontrivial order-3 connected sum, then Σ can be made disjoint from $g(\Sigma)$ and θ is a nontrivial order-2 connected sum by Lemma 2.4. This completes the proof of the Main Theorem.

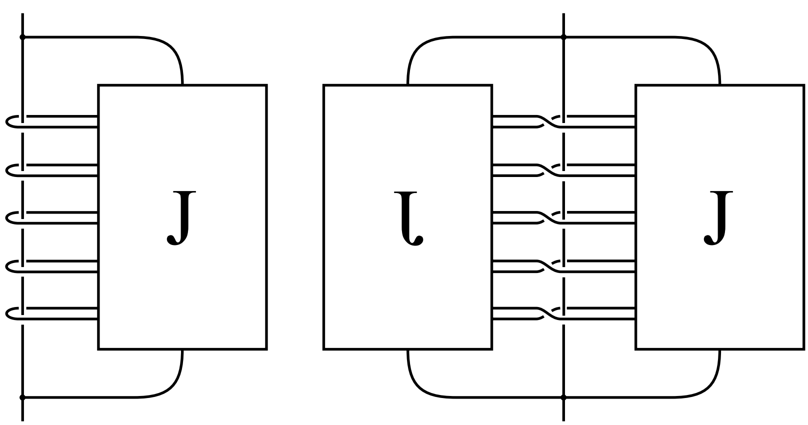
3. Application

To employ the Main Theorem, we must produce the lifted knot for a given theta-curve. Fortunately, this is not difficult. Given a theta-curve $\theta = \kappa \cup e$ with unknotted constituent knot κ, draw the lifted knot using the following algorithm:

(3.1) Draw θ in $\mathbb{R}^3 \cup \{\infty\}$, so that κ comprises the z-axis and the point at infinity. Consider the diagram given by projecting θ onto the zy-plane.

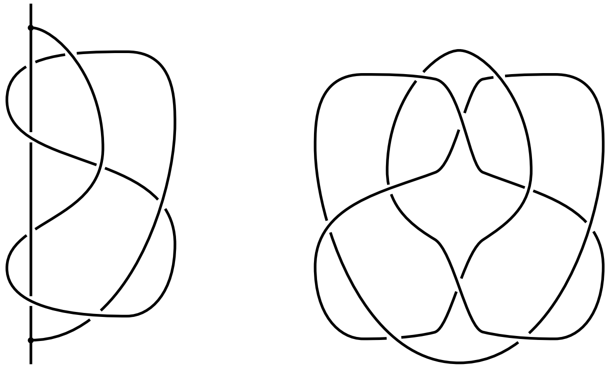
(3.2) By an ambient isotopy fixing κ, arrange for all self-crossings of e to have positive y-coordinate. By a further isotopy, we may assume the diagram appears as shown in Fig. 1(a).

(3.3) Now, lifting e yields the knot K given as the union of the following: (i) the diagram J, (ii) a copy of J rotated one half of a revolution about the z-axis and (iii) arcs between (i) and (ii) as shown in Fig. 1(b).



(a) Theta-curve θ (b) Lift of θ to the double branch cover

Fig. 1. Lifting a theta-curve to the double branched cover, branched over the unknotted constituent knot κ pictured as the z-axis and the point at infinity.



(a) Kinoshita's theta-curve (b) Lift knot K of Kinoshita's theta-curve

Fig. 2. Lifting Kinoshita's theta-curve.

Example 3.1. Kinoshita's well known theta-curve [6] is shown in Fig. 2(a). All three of its constituent knots are unknotted. Applying the algorithm to Kinoshita's theta-curve, we obtain the lifted knot K in Fig. 2(b). With K exhibited as a positive 3-braid, it is a simple exercise to isotop K to the standard $(3, 5)$-torus knot (this fact was also observed by Wolcott [9]). As torus knots are prime [1, p. 95], K is prime and the Main Theorem implies that Kinoshita's theta-curve is prime as well.

Remark 3.2. Previously, the authors [2] produced uncountably many isotopically distinct unions of three rays in $\mathbb{R}^3$ with the Brunnian property (namely, all three rays are knotted, but any two of them are unknotted). To achieve this, we used sequences of three-component tangles lying in thickened spheres. Our main tangle A is shown in Fig. 3. We discovered this tangle independently as described in [2]. A key property of the tangle A proved in [2] was that A is irreducible (namely, no sphere separates A into two nontrivial tangles). By taking the thickened sphere containing A and crushing each of the boundary spheres to a point, one obtains

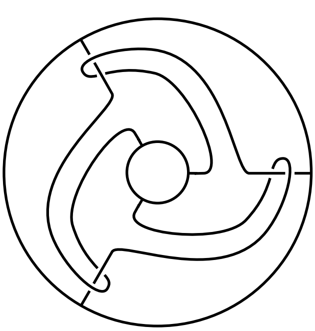


Fig. 3. Tangle A in a thickened sphere.

Kinoshita's theta-curve. As we have just observed, Kinoshita's theta-curve is prime. This immediately implies that the tangle A is irreducible and provides an alternate proof of [2, Theorem 6.1].

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